

Period Doubling Bifurcations and Their Applications to Atomic Force Microscopy:

Summer 2010 Research Summary

By: Daniel Rist

Supervising Professor: Dr. Andrew Dick

In previous semesters work on the identification and characterization of period doubling bifurcations, a macro-scale experimental system was developed to model the basic characteristics of the atomic force microscope. This system let us perform experiments and collect data, which allowed us to determine the natural frequency and damping values of the cantilever beam used in our experimental system. After some adjustment to the clamping mechanism for the beam to get closer to ideal boundary conditions, we were also able to perform impact tests allowing us to obtain bifurcation data and find where the period doubling bifurcations begin for each sample when the cantilever is excited at 2.5 times the natural frequency. The materials that were used in these impact tests were then characterized using a contact resonance method allowing us to obtain the stiffness and damping values for each material used.

With this data collected we can view the relationship between the sample stiffness and the point at which bifurcations begin. These results reveal that the bifurcation point requires more compression as the sample stiffness decreases. However, the data collected is only a very small set of data and does not take into account the effects of stiffness and damping characteristics individually. In order to study this relationship more thoroughly a simulation program is created in order to replicate the characteristics of the macro-scale system.

The simulation model is made up of the sum of the first three spatial mode shapes of the cantilever beam and their time dependent response functions. Because the clamping mechanism on the macro-scale beam did not perfectly provide a clamped boundary condition, the boundary condition used in the simulation was a combination of a pinned boundary condition with a torsional spring. This allows for the torsional stiffness constant to be tuned in order to match the natural frequency of the macro-scale beam. The first three mode shapes are shown in Figure 1 where K is the torsional stiffness constant.

The system model is numerically integrated by using a fourth order Runge-Kutta method. The simulations were able to qualitatively recreate the period doubling bifurcations showed in the macro-scale

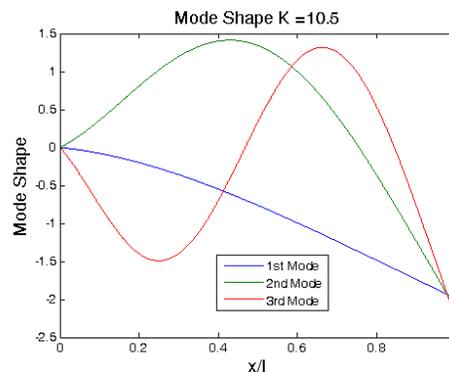


Figure 1: First three mode shapes used to model macro-scale impactor system for numerical simulations.

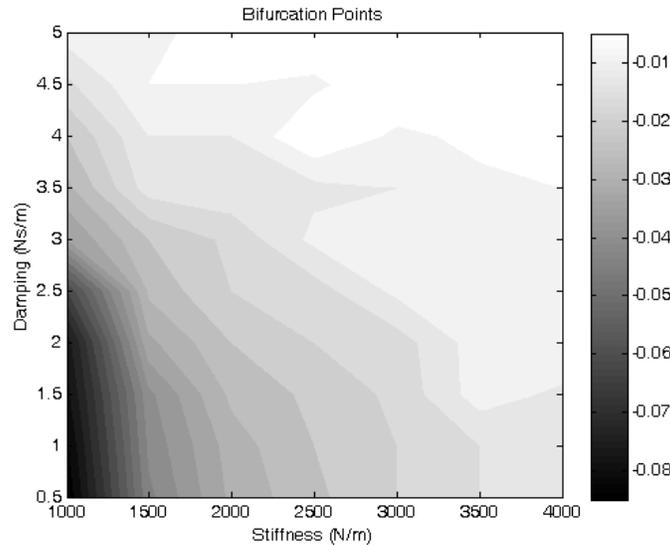


Figure 2: Contour plot illustrating the separation distance of the bifurcation point as a function of both contact material stiffness and damping.

experiments. However, the response amplitude observed in the simulations was significantly higher than that of the macro-scale experiments. This is likely due to the use of a linear damping model in the simulations which will be corrected in future work. Despite the difference in amplitude, the simulations appear to match the macro-scale experiments qualitatively and show a similar relationship between sample stiffness and bifurcation point.

Bifurcation points were recorded for simulations using a range of sample stiffness and damping values in order to view the effects of each parameter individually. The resulting plot of the gathered data is shown in Figure 2 where the color bar on the right represents the compression at the bifurcation point in mm.

The bifurcation point requires less compression as both the stiffness and damping values increase. The slope of this change does not appear to be linear, increasing quickly for low values of stiffness and damping and more gradually increasing as the values increase. The contour lines appear jagged in the upper right section of the plot because the compression is so small at the bifurcation point and it is difficult to differentiate such small values.

Future work on this project will include the addition of a nonlinear damping model to the simulation program. This is expected to improve the agreement between the simulated and the experimental response amplitudes. Also, an electromagnetic component will be added to the macro-scale system in order to create a region of attractive force such as is experienced by an actual AFM cantilever due to the attractive forces of the atoms with which it is interacting. With these improvements made on our system we can begin to find a more exact relationship between the stiffness, damping, and bifurcation point.